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Our Schools Must Teach How to Think!

It would appear that no one doubts this. But would everyone be able to give a direct answer to the directly posed question: what does this mean? What does it mean "to think" and what is "thinking?" A far from simple question and in a certain sense a tricky one. It is worth digging a little deeper to see how this comes to light.

Very often—much more often, perhaps, than it seems—we mix up two very different things here, especially in practice: the development of the ability to think and the process of the formal mastering of the knowledge specified in curricula. By no means do these two processes automatically coincide, although one without the other is also impossible. "Much knowledge does not train the mind," although "lovers of wisdom must know much"—these words, spoken over 2,000 years ago by Heraclitus of Ephes, are not out of date even today.

Truly, "much knowledge" in itself does not train the mind—or the ability (or skill) to think. What then does train the mind? And can it be trained (or train itself) at all?

On this score, there exists a far from groundless opinion according to which the mind (the ability to think, "talent," or simply "ability") is "from God" or, in more enlightened terminology, "from nature," from a person's

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parents. Indeed, is it possible to inculcate "mind" into a person in the form of a system of precisely and rigorously formulated "rules" or operational schemas—in short, in the form of a "logic?" We have to conclude that it is not possible. This conclusion is the fruit of experience that finds graphic expression in the international parable of the fool who, encountering a funeral procession, wants "to join you, not steal from you."* It is well known that the best rules and formulas, when drummed into a stupid head, do not make that head cleverer but are themselves transformed into amusing absurdities. This, alas, is only too well known. Hardly anyone will dispute the fact that the teaching of formal logic, introduced into our schools some time ago "on the personal instructions of Comrade Stalin," did not increase the number of "clever" people or reduce the number of "stupid" people among secondary school graduates.

It is not empirically indisputable experience alone that supports the aforementioned opinion. The most precise and rigorous "rules" that constitute "logic" do not and cannot teach the so-called "power of judgment"—that is, the ability to judge whether a given case or given fact falls under given rules. As Immanuel Kant wrote in his *Critique of Pure Reason*, "the school can only proffer to, and as it were graft upon, a limited understanding of an abundance of rules borrowed from the insight of others, but the power of rightly employing them must belong to the learner himself; and in the absence of such a natural gift no rule that may be prescribed to him for this purpose can ensure against misuse. . . . Deficiency in judgment is just what is ordinarily called stupidity, and for such a failing there is no remedy."** This seems to be true. And here is the opinion of another thinker—cited with great sympathy by Lenin as "sharp-witted"—concerning the "prejudice" that "logic teaches how to think": "This is like saying that only by studying anatomy and physiology do we first learn how to digest food and move" (Hegel, *Soch*., vol. 5, p. 2 [retranslated from Russian]; compare also Lenin, *Soch*., vol. 38, p. 75). This is, indeed, a naive prejudice. That is why the introduction of "logic" into the secondary school curriculum could not justify the hopes that some people placed on it.

Evidently, everything remains as it was. Anyone, even "an obtuse or narrow-minded person," can "be trained through study, even to the extent of becoming learned. But as such people are commonly still lacking in judgment, it is not unusual to meet learned men who in the application of their scientific

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**Taskat′ vam, ne peretaskat′* is a customary greeting at a wedding. Having learned the expression in this context, the fool does not understand that it is inappropriate at a funeral and gets beaten.—Trans.

**The author's quotations are from Kant's *Critique*, with minor syntactical adjustment to the Russian text, in the form given in the 1929 Norman Kemp Smith translation, available at www.hkbu.edu.hk/~ppp/cpr/.—Trans.

knowledge betray that original want, which can never be made good." So Kant sadly sums up his argument. And with this too we have to agree.

But in that case what about the appeal that forms the title of this article? Have I myself not proven, by reference to highly respected authorities, that this slogan cannot be realized and that intelligence is a "natural gift" and not an acquired skill?

Fortunately, this is not so. It is true that the ability (or skill) to think cannot be "grafted" into the brain in the form of a collection of "rules," formulas, and—as people like to say nowadays—"algorithms." A human being is still a human being, much as some would like to turn him into a "machine." In the form of "algorithms" you can "insert" into the skull only a mechanical, that is, a very stupid "mind"—the mind of a cashier, but not the mind of a mathematician.

However, the arguments cited above by no means exhaust the position even of Kant, let alone that of a materialist. First, it is not true that intelligence is a "natural" gift. For his mind, or his ability to think, man owes just as little to Mother Nature as he does to God the Father. To nature he owes only his brain—the organ of thinking. As for his ability to think with the aid of this brain, it not only "develops" (in the sense of "improves") but also first *emerges* only together with his attachment to social-human culture, to knowledge. The same goes for his ability to walk upright, which man likewise does not get "from nature." This is the same kind of "skill" as all the other human abilities. True, while any mother easily teaches her child how to use his rear limbs to walk upright, far from every professional pedagogue is able to teach him how to use his brain for thinking. But a reasonably intelligent and attentive mother does this much better, as a rule, than any other kind of pedagogue. She will never shrug off the difficult effort associated with training the "mind" of her young child on the pretext—so convenient for the mentally lazy "educator"—that the child in question is "naturally" or "congenitally" incapable. The young child is taught "thinking" by all life around him—by his family, by games, by the courtyard, and by other young children like himself, whether they are older or even younger. Caring for his little brother also requires as well as develops "intelligence."

The idea of the "congenital" or "natural" origin of the ability (or "inability") to think is merely a veil that conceals from the mentally lazy pedagogue those real (very complex and individually variable) conditions and circumstances that in fact stimulate and form the "mind," the ability to "think independently." This idea usually serves merely to justify our lack of understanding of these conditions and lazy reluctance to examine them and take on the hard work of organizing them. By shifting the blame onto "nature," we preserve a clear conscience and keep up scientific appearances.

Theoretically, such a position is incompetent; morally, it is vile, because it is extremely antidemocratic. Nor is it in accord with the Marxist-Leninist understanding of the problem of "thinking," or with the communist attitude to man. In terms of natural endowment we are all equal—in the sense that 99 percent of people enter life in this world with a biologically normal brain capable in principle—with a little less or a little more difficulty—of mastering all of the "abilities" developed by their predecessors. And it ill behooves us to dump onto nature the sins of society, which until now has been less just and democratic than nature in distributing its "gifts." It is necessary to open up each person's access to the conditions of human development, including the conditions for the development of the ability to "think independently" as one of the chief components of human culture. And the school is obliged to do this. Intelligence is not a "natural" gift. It is society's gift to a person. It is, incidentally, a gift that he will later repay a hundredfold—from the point of view of a developed society, the most "profitable" of "capital investments." An intelligently organized—that is, a communist—society can be constituted only by intelligent people. And never for a minute must we forget that it is precisely the people of the communist future who are sitting behind school desks today.

The mind, the ability to think independently, takes form and develops only in the course of individual assimilation of the intellectual culture of the epoch. Properly speaking, the mind is none other than this intellectual culture, transformed into a personal possession and legacy, into the principle of a person's activity. "Mind" is made up of nothing else but this. To use the high-flown language of philosophy, it is the individualized spiritual wealth of society.

And this, to put it simply, means that mind (intelligence, talent, ability, etc.) is the natural state of man, the norm and not the exception, the normal result of the development of a biologically normal brain under normal—human—conditions.

Conversely, the "stupid" person, the person with an incorrigible deficiency of "powers of judgment," is above all a maimed person, a person with a crippled brain. And this "crippling" of the organ of thinking is always the consequence of "abnormal" and "unnatural" (from the point of view of the true criteria of human culture) conditions, the result of crudely coercive "pedagogical" influences on this tender organ (especially at an early age).

The organ of thinking is much more easily crippled than any other organ of the human body. And it is very difficult—after a certain age, quite impossible—to mend. To cripple it is simple—by means of a system of "unnatural" "exercises." And one of the most reliable methods of such crippling of the brain and intellect is the *formal memorization of knowledge*. It is precisely by this method that "stupid" people are produced—that is, people with an atrophied power of judgment. People who are unable competently to relate the general knowledge they have mastered to reality and who therefore make a mess of things.

"Cramming," backed up by endless "repetition" (which should be called not the mother but rather the stepmother of learning), cripples the brain and intellect. Paradoxically enough, the truer and "cleverer" the truths inculcated by cramming the more crippling the effect. The point is that a stupid and nonsensical idea from the child's own head will soon be dispelled by experience: when such an idea clashes with facts the child will be forced to doubt, to compare, to ask why, and—in general—to "rack his brains." An "absolute" truth, by contrast, will never give him occasion to do those things. "Brain-racking" of any sort is counterindicative of absolutes: they are motionless and crave only more and more "confirmations" of their infallibility. It is for this reason that an "absolute truth" crammed without understanding becomes for the brain something like a track for a train or blinkers for a workhorse. The brain grows accustomed to move only along beaten (by other brains) tracks. Anything that lies to the right or left of those tracks is no longer of interest to it. It simply no longer pays attention to other things, regarding them as "inessential" and "uninteresting." This is what the prominent German writer B. Brecht had in mind when he said: "A person to whom it is self-evident that two times two makes four will never be a great mathematician" [retranslated from Russian].

Everyone knows what an agonizing experience this crudely coercive operation on the brain—"cramming" and "grafting"—is for any lively child. Only very unpleasant childhood memories could inspire adults to invent these poetically expressive terms. It is not by chance or by caprice that the child experiences "grafting" as violence. The point is that nature has arranged our brain so cleverly and so well that it has no need of any "repetition" or special "memorization" to learn anything that it finds directly "understandable," "interesting," and "useful." So it is necessary to graft only what is incomprehensible, uninteresting, and useless—what has no resonance or counterpart in, and does not "flow" from, the individual's direct life experience.

As numerous experiments have proved, man's "memory" stores everything that has been of concern to its possessor throughout his life. However, some knowledge is stored in the brain, so to speak, in an active state, "within easy reach," and in case of need can always be called into the light of consciousness by an effort of the will. This is knowledge that is closely connected with the sense- and object-oriented life activity of man. This "active" memory is reminiscent of a well-organized workspace in which the craftsman takes hold of the object, instrument, or material he needs without a glance and without specially "recalling" which muscle he has to move for this purpose. It is quite another matter with knowledge that the brain has absorbed in complete isolation from its main activity and placed, so to speak, "in reserve." French psychologists, for example, applied special techniques to the brain of an old semiliterate woman to force her to declaim for hours on end ancient Greek verses of which she understood neither the content nor the meaning and that she "recalled" only because once, many years before, some diligent gymnasium student had memorized these verses aloud in her presence. In the same way, a stonemason "recalled" and accurately drew on paper the fantastically intricate twists and bends of a crack in a wall that he had once had to repair. In order to "recall" things of this kind, a person has to make agonizing exertions and these very rarely succeed. The problem is that the brain submerges an enormous mass of unneeded, useless, and "nonoperational" information in special "dark storerooms" below the threshold of consciousness. Everything that a person has seen or heard at least once is stored there. In special—abnormal—cases, all the junk that has accumulated in these storerooms over many years breaks through to the surface of the higher regions of the cerebral cortex, into the light of consciousness. Then the person suddenly recalls a mass of trivial details that had apparently been long and finally "forgotten." But this occurs precisely when the brain is in a state of passivity, usually that of a hypnotic trance, as in the experiments of the French psychologists. The point is that "forgetting" is not a defect. Quite the reverse: "forgetting" is produced by special wise mechanisms of the brain that protect the organ of thinking (the regions of active brain function) from drowning in unneeded "information." It is the natural defensive reaction of the cortex to the threat of meaningless and stupid overload. Should the strong locks of oblivion break open one fine day in the dark storerooms of memory, all the trash accumulated there would gush forth into the higher regions of the cortex and make it incapable of "thinking"—of selecting, comparing, speculating, and "judging."

The fact that "forgetting" is not a minus, not a defect of our mind, but quite the reverse, an advantage, pointing to a redundant "mechanism" that specially and purposively produces it, was graphically demonstrated by the well-known Soviet psychologist A.N. Leontiev at a séance with the no less well-known possessor of "absolute memory" Sh—skii. The test subject was able to "memorize" at one go a list of 100, 200, or 1,000 words and reproduce it at any time thereafter and in any order. After a demonstration of this astonishing ability, he was asked an innocent question. Could he recall among the words imprinted on his memory the three-letter name of a highly infectious disease? There was a hitch. Then the experimenter appealed to the audience for help. And right away it turned out that *dozens* of "normal" people *remembered* what the man with the "absolute memory" could not remember. The word *tif* (typhus) flashed by on the list, and dozens of people with a "relative" memory—quite involuntarily—recorded this word in their memory. The "normal" memory "hid" this little word, like all of the other 999 little words, away in a dark storeroom, "in reserve." But thereby the higher regions of the cortex, which are in charge of "thinking," remained "free" for their special work—including that of purposive "remembering" by tracing chains of logical connections.

It proved just as difficult for a brain with "absolute memory" to function as for a stomach packed full with stones.

This experiment is very instructive. An "absolute"—mechanical—memory is not advantageous but, on the contrary, detrimental to one of the most important and intricate mechanisms of our brain and mind. This is the mechanism that *actively* "forgets" everything that is not of direct use to the performance of the higher mental functions, everything that is not connected to the logical flow of our thoughts. The brain tries to "forget" what is useless, what is not connected with active thinking, to sink it to the bottom of the subconscious, in order to leave the conscious "free" and ready for the higher forms of activity.

It is this "natural" brain mechanism, which protects the higher regions of the cortex from aggression, from flooding by a chaotic mass of incoherent information, that "cramming" destroys and cripples. The brain is violently forced to "remember" all that it actively tries to "forget," to place under lock and key, so that it should not get in the way of "thinking." Raw, unprocessed, and undigested (by thinking) material is "grafted" into the brain, breaking its stubborn resistance.

Marvelously subtle mechanisms created by nature are thereby spoiled and crippled by crude and barbaric interference. And many years later some wise educator dumps the blame on "nature."

With all its might, the "natural" brain of the child resists being crammed with undigested knowledge. It tries to rid itself of the food that it has not chewed over, to sink it to the lower regions of the cortex, to "forget"—and over and over again it is schooled by "repetition," coerced and broken, using both the stick and the carrot. Eventually the schooling succeeds. But at what a price! At the price of the *ability to think*.

How can we not recall here the surgeons from *The Man Who Laughs*? The pedagogue-comprachicos impose a permanent fixed "grin" on thinking and make it capable of functioning only in accordance with a rigidly "grafted" schema.* And this is the most widespread method of producing "stupid" people.

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*This refers to Victor Hugo's novel, *L'homme qui rit* (1869). The "man who laughs" wore a fixed grin on his face because he had been abducted as a child by comprachicos (a Spanish neologism for "child buyers")—"surgeons" who make a living by deforming their victims into freaks and then selling them as beggars or for exhibition at carnivals.—Trans.

It is good if the student does not take the scholastic wisdom crammed into him very seriously, if he just "serves out his time." Then they do not manage to cripple him completely, and the real life surrounding the school saves him. Life is always cleverer than a stupid pedagogue.

The hopeless blockheads grow precisely out of the most obedient and diligent "crammers." This confirms that both "obedience" and "diligence" are the same kind of dialectically cunning virtues as all other "absolutes" that at a certain point and under certain conditions turn into their opposites, into defects, some of them incorrigible.

And it has to be said that any lively child (and this *is* "from nature") possesses a very precise indicator that distinguishes "natural" pedagogical influences on his brain from violent, crippling ones. He either absorbs "knowledge" with greedy and lively interest or displays obtuse incomprehension and stubborn resistance to violence. He either easily—at one go—"gets the point," showing pleasure as he does so, or, on the contrary, fidgets, plays up, and just cannot "remember" apparently simple things.

The morally sensitive pedagogue always pays attention to these "natural" feedback signals, as accurate as the pain that accompanies "unnatural" exercise of the organs of physical activity. The morally obtuse and mentally lazy pedagogue insists, compels, and eventually "gets his own way." The cries of the child's soul are for him empty whims. He simply continues training the child; whether he uses the carrot or the stick makes no difference.

And from this follows a simple conclusion that is as old as the world. It is impossible to teach a child—or, indeed, an adult—anything, including the ability (skill) to think independently, without adopting an attitude of the closest attention to his *individuality*. The old philosophy and pedagogy used to call such an attitude "love." This little word may also be used. It is not so very imprecise, although some admirers of rigorously mathematical thinking will consider such a definition "qualitative" and therefore "unscientific."

Of course, it is also necessary to adopt an intelligent attitude to indications of the child's "inner feeling." It may be that he is fidgeting not because he is bored but because he ate unripe plums the day before. Well, after all, "individuality" is a capricious and mathematically indefinable thing.

But all these are, so to say, ethical and esthetic preliminaries. How then are we to *teach how to think*? Here, of course, love and attention to individuality are not much to go on, although we cannot do without them.

In broad outline, the answer is as follows. We have to organize the process of the mastery of knowledge, the process of the assimilation of intellectual culture in the same way as the best teacher—life—has organized it for thousands of years. Namely, in such a way that in the course of this process the child should be forced constantly to train not only (and even not so much) the "memory" but also the ability independently to solve *tasks* that require thinking in the proper and precise sense of the word—the "powers of judgment," the ability to judge whether or not a given case fits previously mastered "rules" and if not—then what?

Solving tasks is by no means a privilege of mathematics. The whole of the human search for knowledge is none other than an unending process of posing and solving new tasks—questions, problems, difficulties.

And it is self-evident that a person "understands" scientific formulas and propositions only if he sees in them not simply material that he has to cram but, above all, arduously acquired *answers* to quite definite *questions*—to questions that emerge naturally from the midst of life and urgently demand answers.

It is equally clear that a person who has found in a theoretical formula a clear answer to a question, problem, or difficulty that has been troubling him (in which he has been interested) will not forget this theoretical formula. He will not have to "cram" it. He will remember it easily and naturally. And if he does "forget" it, that is no calamity. He will always *derive* it himself when he again encounters a situation-task with the same set of conditions. And that is the meaning of "intelligence."

So it is necessary to "teach how to think" first of all by developing the ability to pose (ask) questions correctly. Science itself began and begins each time with this—with posing a question to nature, with formulating a problem—that is, a task that is insoluble with the aid of already known methods of action, by following known—beaten and trampled—tracks. Each newcomer to the realm of science, child or adult, must start his journey with this, with the sharp formulation of a *difficulty* that is insuperable with the aid of prescientific means, with the precise and sharp expression of a *problem situation*.

What would we say of a mathematics that forced its students to memorize the answers to exercises printed at the back of the book, showing them neither the exercises themselves nor methods for solving them?

However, we often teach children geography, botany, chemistry, physics, and history in just such an absurd fashion. We tell them the answers found by mankind, often without even trying to explain exactly to which questions these answers were given, found, or guessed.

Textbooks and the teachers who follow them too often, alas, start off immediately with quasi-scientific "definitions." But the real people who created science never started with this. They *finished* with definitions. For some reason, however, the child is "led" into science from the opposite end. And then people are surprised that he is unable to "master" general theoretical propositions, and that having "mastered" (in the sense of crammed) them he is unable to relate them to reality, to "life." In this way the pseudo-scientist grows up, the pedant—the person who sometimes knows the entire literature in his field of specialization by rote but *does not understand* it.

Karl Marx gave a good description of this phenomenon a hundred years ago, with reference to the vulgar bourgeois political economist W. Roscher:

> I shall reserve this fellow for a *note.* Such professorial schoolboys have no place in the *text.* Roscher undoubtedly has a considerable—and often quite useless—knowledge of literature, although even here I seem to discern the Göttingen alumnus rummaging uneasily through literary treasures and familiar only with what might be called official, *respectable* literature. But that is not all. For what avails me a fellow who, even though he knows the whole of mathematical literature, yet understands nothing of mathematics?

If only such a professorial schoolboy, by nature totally incapable of ever doing more than learning his lesson and teaching it, of ever reaching the stage of teaching himself, if only such a Wagner were, at least, honest and conscientious, he could be of some use to his pupils. If only he did not indulge in spurious evasions and said frankly: 'Here we have a contradiction. Some say this, others that. The nature of the thing precludes my having an opinion. Now see if you can work it out for yourselves!' In this way his pupils would, on the one hand, be given something to go on and, on the other, be induced to work on their own account. But, admittedly, the challenge I have thrown out here is incompatible with the nature of the professorial schoolboy. An inability to understand the *questions* themselves is essentially part and parcel of him, which is why his eclecticism merely goes snuffling round amidst the wealth of set *answers*" (letter to Ferdinand Lassalle of June 16, 1862; see K. Marks [Marx] and F. Engel′s [Engels], *Soch.,* vol. 25, p. 404 [English translation from www.marxists.org/archive/marx/works/1862/letters/62_06_16.htm]).

This analysis of the "mind" of the pedant is very instructive for pedagogy, for the art of *teaching how to think*.

Science—both in its historical development and in the course of its assimilation by the individual—in general begins with a *question*, whether it is addressed to nature or to people.

But any real question that arises from the midst of life and is insoluble with the aid of predetermined, customary, stereotyped, routine methods is always formulated for the consciousness as a formally insoluble *contradiction*.

Or, to be even more precise, as a "logical" contradiction that is insoluble by purely logical means—that is, by a series of purely mechanical, machine-like operations on previously memorized "concepts" (or, to be even more precise, on "terms").

Philosophy has long made clear that a real "question" that can be solved only through a further investigation of facts always appears as a "logical contradiction," as a "paradox."

Thus, it is only at the place in the corpus of knowledge where there suddenly appears a "contradiction" (some say this, others say that) that, properly speaking, there arise the need and the necessity *more deeply to investigate the object itself*. It is an indicator that the knowledge recorded in generally accepted propositions is excessively general, abstract, and one-sided.

It is precisely for this reason that the mind that has been trained to stereotyped action in accordance with the set prescription of a "typical solution" and that is at a loss when required to find an independent (creative) solution "does not like" contradictions. It tries to avoid or fudge them, returning over and over again to the beaten track of routine. And when in the final reckoning it fails to avoid or fudge a contradiction, when the "contradiction" stubbornly keeps on appearing, such a "mind" collapses into hysteria, just at the point where it is necessary to "think."

For this reason the attitude of a mind to contradiction is a very accurate criterion of its culture—even, properly speaking, an indicator of its presence as intelligence.

Researchers in the laboratory of I.P. Pavlov once performed a very unpleasant experiment on a dog (unpleasant for the dog, of course).

They assiduously induced and developed in the dog a positive salivary reflex to a circle and a negative reflex to an ellipse. The dog was very good at distinguishing these two "different" shapes. Then one fine day they began to rotate the circle within the dog's field of vision in such a way that it gradually "turned" into an ellipse. The dog became agitated and at a certain point collapsed into hysteria. Two rigorously developed and directly opposed conditioned-reflex mechanisms were activated simultaneously and clashed in conflict, "error," or antinomy. For the dog this was unbearable. The point at which "A" turns into "not-A," the point at which "opposites meet" is exactly that point at which the fundamental difference between human thinking and the reflex activity of the animal sharply and clearly manifests itself.

At this point the animal (and also the mind deprived of true "logical" culture) collapses into hysteria, starts to rush about, and falls captive to chance circumstances.

For the mind equipped with true logical culture the appearance of a contradiction is a signal of the emergence of a *problem* that is insoluble with the aid of strictly stereotyped intellectual actions, a signal to activate "thinking"—the independent examination of the "thing" in the expression of which the antinomy has arisen.

It is therefore necessary to train the "mind" from the very start in such a way that a "contradiction" should give it not cause for hysteria but an impulse to independent work, to independent examination of the thing itself—and not only of what other people have said about this thing.

This is an elementary requirement of dialectics. And dialectics is by no means a mysterious art only for mature and select minds. It is the real logic of real thinking—a synonym for concrete thinking. People must be trained in it from childhood.

I cannot but recall here the wise words spoken not long ago by one old mathematician. Deliberating on the causes of the inadequate culture of mathematical (and not only mathematical) thinking among secondary school graduates over recent years, he gave the following extraordinarily accurate characterization of these causes: curricula contain "too much that is finally established," too many "absolute truths." This is precisely why students, grown accustomed to "swallowing the roast grouse of absolute science," are then unable to find their way to objective truth, to the "thing" itself.

This too sounds, as it were, "paradoxical." However, the mathematician's words are as simple as they are true:

> I recall my own schooldays. We were taught literature by a very erudite follower of Belinsky. And we grew accustomed to looking at Pushkin through his eyes—that is, through Belinsky's eyes. Regarding all the teacher told us about Pushkin as beyond doubt, we too saw in Pushkin only what he told us—and nothing more. . . . So it remained until by chance I happened to come across an article by Pisarev. It threw me into confusion. What is this? Everything was turned upside down and still convincing. What was I to do? And only then did I turn my attention to *Pushkin himself*. Only then did I *myself* discover his true beauty and profundity. And only then did I understand—for real and not in scholastic fashion—both Belinsky and Pisarev.

And this, of course, applies not only to Pushkin. How many people have left school for adult life having memorized "indubitable" propositions about Pushkin from textbooks and contenting themselves with that! Naturally, a person who has swallowed his fill of "the roast grouse of absolute science" no longer wants to look at live grouse flying in the sky. After all, it is no secret that very many people had any desire to read Pushkin knocked out of them precisely during literature lessons at secondary school—and not only Pushkin.

It may be objected that our schools are obliged to teach students the "indubitable" and "firmly established foundations" of modern science and not to sow doubts, contradictions, and skepticism in their immature brains. True. But at the same time it should not be forgotten that all these "firmly established foundations" are themselves none other than the results of a difficult search, none other than laboriously acquired *answers* to *questions* that once arose (and are still comprehensible)—none other than *resolved contradictions*.

And not "absolute truths" that fell from the sky into the heads of geniuses. After all, someone must have caught and roasted the roast grouse. And what must be learned in science is how to do this, not how to swallow gruel already masticated by others' teeth. And it must be learned from the very first step—because later on it will be too late.

"The naked result without the road that leads to it is a corpse" [retranslated from Russian], dead bones, the skeleton of truth, incapable of independent movement—thus did the great dialectician Hegel splendidly express himself in his *Phenomenology of Mind*. A set scientific truth recorded in verbal terminology and divorced from the route by which it was acquired turns into a verbal husk even as it retains all the external signs of "truth." And then the dead seizes hold of the living and does not allow it to go forward along the road of science, along the road of truth. Dead truth becomes the enemy of living, developing truth. This is how we get the dogmatic and ossified intellect who at the graduation examinations is awarded a "five" but whom life gives a "two" or even lower.*

Such a person does not like contradictions because he does not like unsolved questions. He likes only set answers. He does not like independent mental labor, preferring to take advantage of the fruits of the mental labor of others. He is a parasitical consumer, not a creative producer. Our schools, alas, have already manufactured many such.

And this is inculcated from childhood, from the first grade. And by those "pedagogues" who like to dump the blame for "lack of ability" on blameless "nature." It is time to drive this vile fable, so convenient for lazy teachers, out of our pedagogical milieu as mercilessly as we drive out the stupid fables of religion.

To teach specifically human thinking means to teach dialectics—the ability rigorously to formulate a "contradiction" and then find its real resolution through the concrete examination of the thing, of reality, and not by means of formal verbal manipulations that fudge "contradictions" instead of resolving them.

Here lies the whole secret. Here lies the difference between dialectical and formal logic, between human thinking and the psyche of any mammal or the actions of a computer. A computer also enters a state of "self-arousal" that very

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*In the Soviet Union and Russia students were and are graded on a five-point scale, five being the highest mark.—Trans.

precisely "models" the dog's hysteria in Pavlov's experiments when two mutually exclusive commands—a "contradiction"—are input simultaneously.

For a human being, by contrast, the appearance of a contradiction is a signal to activate "thinking" and not hysteria. This must be taught from childhood, from a person's first steps in science. Here lies the sole key to the transformation of "didactics" on the basis of dialectical materialism, on the basis of dialectics as the materialist logic and theory of knowledge. Otherwise all talk of such a "transformation" will remain a pious wish, an empty phrase. For the "core" of dialectics, without which there is no dialectics, is precisely "contradiction"—the "motor" and "mainspring" of developing thinking.

There is nothing especially "new" here. Any reasonably intelligent and experienced pedagogue does and has always done this. Namely, he always tactfully guides the child into a "problem situation," as it is called in psychology—that is, a situation that is insoluble with the aid of methods of action already developed by the child, with the aid of "knowledge" already mastered by him, but a situation that is at the same time well within his capabilities, given his (precisely assessed) existing knowledge. A situation that, on the one hand, requires the active use of all his previously accumulated intellectual baggage, and, on the other hand, does not "yield" completely to him but demands "a little extra"—an argument of his own, an elementary creative device, a drop of "independence" of action.

If after a process of trial and error a person finds a "way out" of such a situation without direct prompting or coaching, he takes a real step along the path of mental development, of the development of "intelligence." And such a step is worth more than a thousand truths mastered in the set form of others' words.

For it is only and precisely thus that a person develops the ability to perform actions that require him to go beyond the *given conditions* of a task.

In this sense, a dialectic exists wherever a person goes beyond that set of given conditions within which the task remains *solved and unsolved* (and therefore has the appearance of a "logical contradiction" between the "goal" and the "means" for attaining it) into that broader set of conditions within which it is really—concretely, in relation to objects, and therefore "obviously"—*solved*.

Such a dialectic is realized even in the case of solving a simple geometrical task requiring a transformation of the conditions given by the initial diagram—even should this transformation consist only in drawing the one and only "extra" line that joins two other (given) lines that were previously unjoined, unconnected, or—the term used in logic—"unmediated." The line that accomplishes the *connection*—the transition, the conversion—and therefore incorporates the characteristics of the two lines that it connects, both "A" and "not-A."

In this way we resolve, in object-oriented action and in contemplation, the situation that brought the dog to a state of hysteria. The situation of transition or conversion of one clearly defined form to another—of a circle to an ellipse, of a polygon to a circle, of a straight line to a curve, of an area to a volume, and so on and so forth—in general, of "A" to "not-A."

Any task that requires such a transition from the given or known to the unknown always entails the conversion of fixed opposites.

If "A" is known to us (its qualitative or quantitative characteristics or "parameters" are given) and we need to find "B"—that is, express "B" through the characteristics of "A"—and do not as yet know this 'B," then this means that for the time being we can say only that it is "not-A." But what is it apart from being "not-A"?

It is for this that we need to find a *transition* or "bridge." The transition from one thing to a second—from "A" to "not-A"—can in general be accomplished only through a "mediating link," through a "middle term of the deduction," or—as it is called in logic—through "a third."

Finding such a middle term is always the chief difficulty of a task. It is here that the presence or absence of "sharp-wittedness," "resourcefulness," and other qualities of the "intelligent mind" comes to light.

This unknown "third" always possesses clearly marked dialectical properties. Namely, it must incorporate simultaneously the characteristics of "A" and the characteristics of "B" (that is, "not-A").

For "A" it must represent "B" and for "B" it must be an image of "A."

In the same way, a diplomat in a foreign country "represents" not himself but his country. In country "A" he is a representative of "not-A." He must speak in two languages, in the languages of both countries—in that of the country *that* he represents and in that of the country *in which* he is a representative.

In other words, the "middle term" must directly combine within itself the characteristics of the sides of the contradiction that it "mediates"—both "A" and "not-A." It is a direct unity of opposites—the point at which they *turn* into one another.

For so long as the "sides of the contradiction" are not mediated—that is, there is "A" and, alongside it, "not-A"—we have a *logical contradiction*. A logical contradiction is an unmediated, unresolved contradiction. In this sense—in the sense that it expresses an *unsolved* question—it is something "intolerable."

To solve a question means to find that "third" by means of which the initial sides of the contradiction, "A" and "not-A," are joined, connected, and are expressed through one another—that is, *turn* (in thinking) into one another. This is the same situation that they created for the dog in Pavlov's laboratory by "turning" a circle into an ellipse.

But what significance does this have for the movement of thought, for training the ability to "think?" Enormous significance.

Above all, if we have clearly registered the conditions of a task as a "contradiction," then our thinking is aimed at seeking out that fact (line, event, action, etc.) solely by means of which the initial contradiction can be resolved.

For the time being we do not know what this third is. This is what we must seek and find.

But at the same time we do already know something extraordinarily important about it. Namely, it must simultaneously fit the characteristics of "A" and the characteristics of "B" (that is, of "not-A"). The search for the "middle term" of the deduction is therefore *goal directed*. It must be a real fact that, expressed through the terms of the initial conditions of the task, will look like "A" and like "not-A" simultaneously and in the "same relation"—as a "contradiction."

From the point of view of purely formal thinking, such a fact seems something quite impossible and unthinkable. Yes, it is "unthinkable" in the sense that it is not as yet present in our thinking and in the field of our contemplation—*in the given conditions of the task*. But, after all, in the final analysis all progress in our knowledge comes down to bringing what was previously "unthinkable" within the ambit of our thought: we find, see, and comprehend. And thereby we resolve a previously unresolved task, question, or contradiction.

Dialectics consists in formulating a "contradiction," bringing it to the fullest sharpness and clarity of expression, and then finding a real, concrete, object-related, and therefore obvious, resolution of it. And this is always accomplished by discovering a new fact in the context of which the "contradiction" previously exposed by us is simultaneously *realized* and *concretely resolved*.

A sharply formulated contradiction creates a "tension of thought" that is not released until the fact solely by means of which it is resolved is found.

This occurs in both the most complicated cases of intellectual development and in the simplest. It was precisely dialectics that enabled Karl Marx to solve a problem over which bourgeois economists had racked their brains in vain—the problem of the emergence of capital from the exchange of commodities. First of all, a sharp contradiction was registered here. The trouble is that the supreme law of market relations is the exchange of equivalents, of equal values. If I have an object worth 5 rubles, I can exchange it for other commodities that are also worth 5 rubles. I cannot by means of exchange—by means of a series of sales and purchases—turn 5 rubles into 20 (if, of course, we exclude speculation and deception). But how then are profit, surplus value, and capital possible? The law of capital is ceaseless "self-expansion." And hence there arises the question:

> Our friend, Moneybags, who as yet is only an embryo capitalist, must buy his commodities at their value, must sell them at their value, and yet at the end of the process must withdraw more value from circulation than he threw into it at starting. His development into a full-grown capitalist must take place, both within the sphere of circulation and without it. These are the conditions of the problem. *Hic Rhodus, hic salta!* [Here is Rhodes, leap here!]. (K. Marks [Marx], *Kapital* [Capital], vol. 1, pp. 172–73 [English translation from www.econlib.org/library/YPDBooks/Marx/mrxCpA5.html])

So how then—without any deception, that is, without violating the supreme law of the world of commodities—does "capital" suddenly make its appearance—a phenomenon the characteristics of which directly contradict the law of the exchange of equivalents?

The task is posed sharply and clearly. Its solution, Marx continues, is possible only on the condition that "our friend, Moneybags" should "be so lucky as to find, within the sphere of circulation, in the market, a commodity, whose use-value possesses the peculiar property of being a source of value, whose actual consumption, therefore, is itself an embodiment of labor, and, consequently, a creation of value" (Marks, *Kapital,* vol. 1, p. 173 [English translation from www.econlib.org/library/YPDBooks/Marx/mrxCpA6.html]).

A commodity whose *consumption* is a *creation*! A thing that appears to be impossible, "unthinkable"—because it is "logically contradictory."

But if our friend, Moneybags, has nonetheless turned himself into a capitalist, then he has indeed solved the problem that is insoluble from the point of view of the supreme law of the world of commodities. He has exchanged kopeck for kopeck in the most honest fashion, never swindling a single soul, and still ended up with a ruble. And this means that he has found and bought in the market that unthinkably marvelous object—a commodity-value the consumption (destruction) of which is identical to the production (creation) of value.

And for the theorist to resolve the theoretical (logical) contradiction he then has only to investigate where Moneybags contrived to buy such a highly original commodity with the aid of which the unthinkable becomes "thinkable." And what is this magical object that accomplishes the unthinkable without violating in the slightest the strict law of the world of commodities? The author of *Capital* follows him and discovers that "the possessor of money finds on the market such a special commodity in capacity for labor or labor power" (Marks, *Kapital,* vol. 1, p. 173).

This is the sole commodity in the market that enables us to resolve the contradiction that no trick with terminology is capable of resolving. This is the sole object that is strictly subordinate to all laws of the "commodity" and strictly fits all theoretical definitions of "commodity" and "value" (those same definitions and laws from the point of view of which the birth of capital is an "unlawful" act) and that at the same time, in the strictest accordance with law, gives birth to this "unlawful" offspring—surplus value and capital, that is, phenomena that directly contradict the laws of the world of commodities. This is just such an object in the very existence of which the conversion of "A" to "not-A"—of use value to exchange value—is accomplished. A "conversion" that is just as natural—and at the same time just as "unbearable" for nondialectical thinking—as the conversion of the circle to the ellipse, to the noncircle.

The required fact has been found—the directly real, concrete, and obvious fact—and the "logical contradiction" that is otherwise *insoluble* has been resolved.

And here we can see very clearly that it is precisely the "logical contradiction" exposed within the initial conditions of the task and within those conditions unresolved and insoluble that provides thinking with those conditions to which the "unknown"—the "X" or missing link that we have to find to rigorously solve the task—must correspond.

And the more sharply the "contradiction" is formulated, the more precisely indicated the "signs" to which this "unknown" must correspond, the criteria in accordance with which the search must be guided and attention directed. In this case a person's thinking does not wander here and there in the hope of stumbling across a new fact, but *purposefully seeks out that fact*—the unique fact that will enable him to close the chain of reasoning.

Figuratively we can picture this mechanism of dialectical thinking as follows. It brings to mind a severed electric wire. At one end of the wire a positive charge has accumulated, at the other a negative charge. The tension between the two opposed charges can be released only by using some object to close the circuit. What kind of object? Let us experiment. We connect the ends of the wire with a piece of glass. Current does not flow; the tension remains. We try wood. The result is the same. But as soon as we place a piece of metal in the gap between the poles current flows and the tension is released.

The "tension of contradiction" in thinking is released in a similar fashion—by inserting a new fact into the chain of reasoning that has been "severed" by the contradiction. Not, of course, just any fact that happens to come to hand, but only the unique fact that "fits" the conditions of the task and connects or "mediates" the previously "unmediated" sides of the contradiction. It must be a fact that simultaneously "fits" the characteristics (lawful requirements) of both sides of the contradiction.

For side “A” it must be a “representative” of side “B” (that is, “not-A”), while for side “B” it must be a representative image of side “A” (which is, of course, “not-B”). Otherwise it could not be a “conductor” or “intermediary” between them, just as an interpreter between two people who speak different languages can only be a third person who speaks both languages. It must possess within itself, as parts of its “specific” character, the indicators of both “A” and “B”—that is, it must be a direct combination (unity) of different and opposite attributes.

Once we have found such a fact, the “contradiction” ceases to be “unmediated” and unresolved. For so long as we have not found it, the contradiction remains an unresolved “logical” contradiction and creates the very “tension of thought” that gives us no rest until the task is solved.

To acquire the culture of thinking means, therefore, to learn to “bear the tension of contradiction” and not try to avoid or fudge it and if that fails collapse into hysteria, rage, and irritation. On the contrary, we must always tackle a contradiction head on and try to reveal it in its “pure form” in order then to find its concrete, object-related, and obvious resolution in facts.

Dialectics consists in bringing to light in facts, in the set of facts that constitute the system of conditions of the unsolved task, their own contradiction, in lending this contradiction the utmost clarity and purity of expression, and then in finding its “resolution” again in facts—in the unique fact that is not yet in the field of view and that needs to be found. The contradiction itself compels us to seek out such a fact. In this case, the contradiction in thinking (i.e., the “logical contradiction”) is resolved in the same way that reality, the movement of the “thing itself” resolves real contradictions.

And not by means of purely terminological manipulations, not by “clarifying concepts” and their definitions.

(Of course, no objection can be made against the striving to “clarify concepts.” By checking and rechecking the preceding course of reasoning that has led to the “logical contradiction” we very often discover that this contradiction is merely a consequence of simple carelessness, ambiguity in terms, or some similar cause, and therefore does not express any real object-related problem. Contradictions of this kind are resolved by purely formal means—namely, by “clarifying concepts”—and require no search for new facts.

However, dialectics requires formally impeccable thinking. What is said above applies only to those “logical contradictions” which emerge in reasoning as a result of the most rigorous and formally impeccable thinking, of thinking that gives logical expression to the real conditions of the task. This must be borne in mind.)

It is for this reason that the highest culture of thinking, the ability to bear the “tension of contradiction” without irritation or hysteria, the ability to resolve a contradiction in reality and not in words always finds expression in knowing how to argue with oneself. What distinguishes a person who thinks dialectically from a person who thinks undialectically? The ability to weigh up all the "pros" and all the "cons" on your own, without the presence of an "external opponent," without waiting until an adversary with malicious joy shoves those "cons" in front of your nose.

A person with cultured thinking is therefore always very well prepared for disputes. He has foreseen and weighed up all the "cons" and has his counterarguments ready.

The person who in preparing for a dispute confines himself to collecting with diligent partiality "pros" and "confirmations" of a noncontradictory thesis is always beaten. He is beaten from angles that he has not anticipated. And the more diligently he has sought out "confirmations," the more diligently he has closed his eyes to the real "sides" of a thing that may provide grounds for an opposing view, the more such angles there are.

In other words, the more one-sided (the more abstract and general) the, for him, "indubitable" thesis that for some reason he prefers, the more "indubitable" and "absolute" the truth that he has memorized and mastered as an internally "noncontradictory" thesis.

It is here that all the cunning of "absolute truths" manifests itself. For the more "absolute" and "certain" a truth, the closer it approaches the fateful point of transformation into its own opposite, the easier for an opponent to turn it against itself, the more facts and evidence can be cited against it.

Two times two is four?

Where did you see that? In very rare cases, artificial and exceptional cases. In cases involving only solid, mutually impenetrable bodies. Two drops of water "added" together will yield only one drop, or perhaps twenty-one. Two liters of water "added" to two liters of alcohol will never give you four liters of vodka, but always a little less. And in general "two times two is four" would be absolutely infallible only if the universe consisted solely of "absolutely solid bodies." But do such bodies really exist, at least by way of exception? Or do they, perhaps, exist only in our own heads, in idealizing fantasy? Not an easy question. Atoms and electrons, in any case, are not such bodies.

It is precisely for this reason that those mathematicians who are convinced that their statements (mathematical truths) possess "absolutely indubitable" universality are inclined toward the idea that these statements do not and cannot reflect anything in the real objective world and that the whole of mathematics from start to finish is merely an artificial subjective construction, the fruit of the "free" creativity of our own spirit and nothing more. And then the fact that these statements are in general applicable to empirical facts and "work" splendidly in the course of their analysis, in the course of the investigation of reality becomes a mystical enigma.

And there you are! Philosophical idealists—as always in such cases.

And that is your punishment for blind faith in an apparently obvious absolute thesis like "two times two is four."

Absolutes are in general not only static but also extremely cunning. He who blindly places his faith in any absolute as something "indubitable" will sooner or later be vilely betrayed by it. Like that dog who was trained thoughtlessly to salivate at the sight of a circle.

So is it really appropriate to inculcate in the young child a blind trust in such patent traitors? Is this not deliberately to prepare him as a burnt offering, as a sacrifice to "absolute truths"—instead of educating him to mastery over them?

A person brought up to regard "two times two is four" as an indubitable truth over which it is impermissible even to ponder will never even become simply a mathematician, let alone a *great* mathematician. He will not know how to conduct himself in the sphere of mathematics as a human being.

In this field he will always remain merely a guinea pig whom the teacher will constantly present with highly unpleasant and incomprehensible surprises like the conversion of a circle into an ellipse, of a polygon into a circle, of a curve into a straight line and back again, of the finite into the infinite, and so on and so forth. He will perceive all these tricks as black magic, as the mysterious art of mathematical gods whom it is necessary only to adore and worship blindly.

But life will show him not only how two times two turns into five, but also how it turns into a wax candle. Life is full of change and transformation at every turn. Little in it is absolutely static. Science for him will be only an object of blind worship, while life will abound in occasions for hysteria. The connection between science and life will always remain for him mystically incomprehensible, beyond his grasp and reach. Life will always appear to him as a quite "unscientific" and even "irrational" thing, and science as a vision that soars over life and bears no resemblance to it.

The "grafting of absolutes" onto the brain of the young child can have no other outcome. The stronger and blinder the faith that a person places in their infallibility as a child, the more cruelly life will punish him with disillusionment in science, lack of faith, and skepticism.

For in any case he will not evade contradiction—the conflict between a general idea or abstract truth and the diversity of living facts that it does not accommodate. Sooner or later he will be drawn into such a conflict—and will be compelled to *resolve* the contradiction. And if you have not taught him how to do this, if you have convinced him that the truths impressed upon him are so absolute and indubitable that he will never come across a fact that "contradicts" them, then he will see that you have deceived him. And then he will cease to believe both in you and in the truths that you have drummed into him.

Philosophy and psychology established long ago that the "skeptic" is always a disillusioned "dogmatist," that "skepticism" is the reverse side of "dogmatism." These are two mutually reinforcing positions, two dead millstones between which a stupid education grinds the living mind.

The training of a dogmatist consists in teaching a person to look at the world around him only as a reservoir of "examples" that illustrate the correctness of one or another abstract general truth. At the same time he is carefully shielded from contact with facts that favor the opposing view, and above all he is prevented from reading works that defend this opposing view. It is self-evident that only a mind quite incapable of a critical attitude toward itself can be trained in this fashion. It is equally obvious that such a hothouse-grown mind can survive only under a bell-glass, in sterile conditioned air, and that spiritual health thus preserved is just as fragile as the physical health of a child kept indoors out of fear that he will catch a cold. Even the slightest breeze will ruin such health. The same thing happens to a mind that is carefully shielded from encounters with the contradictions of life, a mind that fears works that question the dogmas it has memorized.

It is of much greater benefit to "the good cause"—Kant writes in the *Critique of Pure Reason*—to study counterarguments than it is to read works that demonstrate what you already know. "The reply of the *dogmatic* defender of the good cause," he continues, "I should not read at all. I know beforehand that he will attack the sophistical arguments of his opponent simply in order to gain acceptance for his own; and I also know that a quite familiar line of false argument does not yield so much material for new observations as one that is novel and ingeniously elaborated. . . .

But must not the young, at least, when entrusted to our academical teaching, be warned against such writings, and preserved from a premature knowledge of such dangerous propositions, until their faculty of judgment is mature, or rather until the doctrine that we seek to instill into them has taken such firm root, that they are able effectively to withstand all persuasion to contrary views, from whatever quarter it may come?"

This seems reasonable, Kant says. But . . .

"But when, later, either curiosity or the fashion of the age brings such writings under their notice, will their youthful conviction then stand the test?"

Doubtful. For the person who is accustomed only to the dogmatic cast of mind and does not know how to develop the dialectic hidden in his own soul no less than in the soul of his adversary, the opposing conviction will possess "the advantage of novelty," while the familiar conviction, learned with "the credulity of youth," has already lost this advantage.

"And accordingly he comes to believe that there can be no better way of showing that he has outgrown childish discipline than by casting aside these well-meant warnings; and accustomed as he is to dogmatism, he drinks deep draughts of the poison, which destroys his principles by a counter-dogmatism."

All this, of course, remains true even today. This is a psychological law that has its prototype in the logic of things.

It is precisely for this reason that Hegel considered "skepticism" a higher level of development of the spirit than "dogmatism"—the natural form of the overcoming of naive dogmatism.

For while the dogmatist stubbornly defends "half of the truth" against the other "half of the truth," not knowing how to find the "synthesis of opposites" or "concrete truth," the skeptic—who also does not know how to accomplish this concrete synthesis—at least *sees both halves,* understanding that there are grounds for both, and wavers between them.

The skeptic therefore has a chance of seeing the "thing" on which dogmatists break their lance as a "unity of opposites"—as that unknown "third" that appears to one dogmatist as "A" and to another as "not-A."

And two dogmatists—like two rams on a bridge—are doomed to eternal strife. They will butt one another until they both fall into the cold water of skepticism.

And only after bathing in its sobering stream will they become cleverer—if, of course, they do not choke or drown.

Dialectical thinking, according to Hegel, incorporates "skepticism" as its "inner" organically inherent element. But as such it is no longer "skepticism" but simply rational self-criticism.

A living, dialectically thinking mind cannot be constituted from two equally dead halves—from "dogmatism" and "skepticism." It is, once again, not simply a mechanical combination of two opposite poles but some "third." This third is a combination of rational (and therefore firm) conviction with equally rational (and therefore sharp) self-criticism.

In the eyes of a dogmatist this "third" always looks like "skepticism"; in the eyes of a skeptic it always looks like "dogmatism."

In actual fact, this is *dialectics*—the dialectics of a mind capable of reflecting the dialectics of reality, a logic of thinking in accord with the logic of things.

It is only by keeping all this in view that it is possible to construct a didactics aimed at training a true mind.

And if you want to bring a person up as a consummate skeptic and doubter, then there is no more reliable method of doing this than to inculcate in him a blind trust in "the absolute truths of science"—in the best and truest truths, in those truths that would never deceive him had he learned them not blindly and thoughtlessly *but with intelligence*.

And, conversely, if you want to bring up a person who will not only be firmly convinced of the might of knowledge but will also know how to apply its might to the resolution of the contradictions of life, then measure out for the "undoubter" a dose of "doubt" that will do him no harm—a dose of *skepsis,* as the ancient Greeks called it. Do as physicians have long done, when they inoculate a newborn baby with a weakened vaccine of the most terrible (even for an adult!) diseases. Make him catch these diseases in a weakened, safe form—the form that a person and his mind need. Train him independently to *check* each general truth in eyeball-to-eyeball confrontation with facts that directly contradict it. Help him to resolve the conflict between general truth and specific fact *in favor of authentic, concrete truth*—that is, to the joint benefit of science and fact.

And not to the benefit of "fact" and the detriment of "science," as often happens with dogmatists who have despaired of resolving this conflict rationally and therefore become disillusioned with science and betray it on the pretext that it "no longer corresponds to life."

Then the terrible microbe of disillusionment and skepticism will not lie in wait to poison your student as he crosses the school threshold. Well and truly immunized, he will know how to uphold the honor of scientific knowledge in the event that it comes into conflict with "facts" and "factoids" that "contradict" it. He will know how to interpret these facts scientifically and not by means of the philistine "adaptation" of science to them, not by betraying scientific truths "for the sake of the facts," "for the sake of life," but in reality for the sake of the philistine principle of "such is life."

Only thus is it possible to develop in a person the ability to think, to think *concretely*.

For it is only possible to think *concretely*. Because truth itself is always concrete, because "abstract truth does not exist" (Lenin).

This wise truth, which the greatest minds of humanity—Spinoza, Hegel, Marx, Engels, Plekhanov, Lenin—have not tired of repeating over the centuries, is still, unfortunately, far from becoming a leading principle of our didactics and pedagogy.

True, we very often—too often, perhaps—pay lip service with the word "concrete," squandering this precious concept on trifles to which it has no relation.

Do we not too often confuse "concreteness" with "obviousness"? After all, these are very different things—at least in Marxist-Leninist philosophy, in the logic and theory of knowledge of materialism.

In scientific philosophy "concrete" is by no means understood to mean "obvious." Marx, Engels, and Lenin categorically repudiated the equating of these two concepts as a very bad legacy of medieval scholastic philosophy. For Marx, Engels, and Lenin "concrete" is a synonym of "unity in diversity." That is, the word "concrete" is reserved for a lawfully connected aggregate of real facts, or system of determining facts, understood in their interconnection and interaction.

Where there is no such system, where there is merely a heap or conglomeration of all sorts of "obvious" facts and examples confirming some meager and abstract "truth," there can be no question of any "concrete knowledge" from the point of view of philosophy.

On the contrary, in this case "obviousness" is merely a masquerade costume under which is hidden from people the most cunning and most repulsive enemy of "concrete thinking"—*abstract* knowledge in the worst and most precise sense of the word, in the sense of empty, isolated from life, from reality, from practice.

True, you often hear the following "justification." Up in the higher realms of philosophical wisdom, "concrete" may mean some very complicated things. But didactics is a simpler science. It does not concern itself with the heights of dialectics, and it is thus permitted all that is not permitted to higher philosophy. Therefore, it is not so terrible if we understand by "concreteness" precisely "obviousness" and do not go into excessively fine distinctions.

At first glance this seems right. So what if in pedagogy the term "concrete" is not distinguished very clearly from the term "obvious"? Is it really a matter of terminology? "A rose by any other name smells as sweet." If it were merely a matter of terminology, we could agree with all this. But the trouble is that it is not.

The point is that while it may all begin with confusion over terms, the confusion to which this leads is no joking matter.

In the final analysis, "obviousness" (the principle in itself is neither good nor bad) is not the ally and friend of true (= concrete) thinking that the didacticians think it must be, but something quite the reverse. It is precisely that masquerade costume under which is hidden the most abstract—in the worst sense—thinking and knowledge.

Combined with true *concreteness*, "obviousness" is a mighty means of developing a thinking mind.

But combined with *abstractness*, the same "obviousness" becomes a reliable means of crippling and perverting the child's mind.

In the one case it is a great blessing, in the other an equally great evil—just as rain may benefit the harvest in one case and harm it in another.

And when teachers forget this, when they start to see "obviousness" as an absolute and unconditional "blessing," as a panacea for all evils, and above all for bad "abstractness," for the formal verbal assimilation of knowledge, it is then that they unknowingly render the greatest service to the enemy—the "abstract." They hospitably throw open to him all the doors and windows of the school, provided that he has the wit to appear there in the masquerade costume of "obviousness," under a cloak decorated with little pictures, "graphic textbooks," and the other attributes that make up his "concrete" camouflage.

And that is terrible. An open enemy is much to be preferred over an enemy who passes as a friend.

That is where the confusion leads.

First let me tell a wise parable made up 150 years ago by a very clever man. This parable is titled: "Who thinks abstractly?" Here is the first part.

> A murderer is being led to execution. For the crowd of onlookers he is a murderer and nothing more. It may so happen that ladies who are present observe, among other things, that he is a fine figure of a man, even a handsome man. The crowd finds this a reprehensible remark: "What? The murderer is handsome? How can you think such a terrible thing? How can you call a murderer handsome? You yourselves, I dare say, are no better!" And perhaps a priest, in the habit of looking deep into things and into human hearts, adds: "This is a sign of the moral corruption that reigns in the highest circles of society."
>
> The connoisseur of people takes a different approach. He traces the course of events that shaped the criminal and discovers in the story of his life and upbringing the influence of parental discord in his family. He sees that once this person was punished with excessive severity for a trifling offense; this has embittered him, inclined him against the legal order, and aroused his resistance, placing him outside society, so that eventually crime has become his sole means of self-affirmation.
>
> The crowd, were they to hear this, would surely be indignant: "He wants to justify a murderer!"
>
> I recall how in the days of my youth there was a mayor who complained that writers had sunk so low as to undermine the foundations of Christianity and the legal order: one of them, heaven forbid, even defended suicide!
>
> Further explanation by the shocked mayor made it clear that he was speaking of [Goethe's] *The Sufferings of Young Werther*.

This is what is called *thinking abstractly*—seeing nothing in a murderer beyond the abstraction that he is a murderer, and by means of this simple quality extinguishing all other qualities of the human being in the criminal.

But let us proceed to the second part of the parable.

> "Hey, old woman, you're selling rotten eggs," the shopper says to the trader. "What?" the trader bursts out. "My eggs are rotten? You're rotten yourself! You dare to tell me such a thing about my wares? And who are you? Your dad was eaten alive by lice and your mum had affairs with Frenchmen! You, whose grandma snuffed it in an almshouse! Look, you've twisted a whole bedsheet into your shawl! Never fear, everyone knows where all this stuff came from! If it weren't for the officers, you and your kind wouldn't be parading around in finery! Decent women take better care of their homes, but the place for you and your kind is in jail! You'd be better off darning the holes in your stockings!" In short, she cannot make allowance for the tiniest drop of good in the woman who has insulted her. [retranslated from Russian; exact source not given in original]

She too is *thinking abstractly*, summing up everything, starting with the shawl and ending with the stockings, from head to toe, and throwing in the shopper's dad and other relatives for good measure, exclusively in the light of her crime in saying that the trader's eggs were not fresh. She views everything through the prism of these rotten eggs, whereas those officers to whom she refers—if, of course, they have anything to do with the matter at hand, which is very doubtful—would prefer to notice quite other things in a woman.

This parable does not seem to need lengthy commentary. Its author—the great dialectician Hegel—uses it to illustrate a very simple and deeply true, albeit at first glance paradoxical, proposition: "Who thinks abstractly? The uneducated person, and by no means the educated one."

The person of intellectual culture never thinks abstractly because that is too easy, by reason of "the inner emptiness and pointlessness of this pastime." He is never contented with a meager verbal definition ("murderer," etc.), but always tries to examine the thing itself in all its "mediations," connections, and relations, and, moreover, in development causally conditioned by the entire world of phenomena that has produced this thing.

It is thinking of this kind—cultured, competent, and flexible *object-oriented* thinking—that philosophy calls "concrete thinking." Such thinking is always guided by its own "logic of things" and not by any narrowly selfish (subjective) interest, prejudice, or aversion. It focuses on the objective characteristics of a phenomenon, aiming to reveal their necessity—that is, the law that governs them, and not on trivial details that happen to catch the eye, be they a hundred times more "obvious."

Abstract thinking is guided by general words, by memorized terms and phrases, and therefore sees very little of the wealth of real phenomena. It sees only what "confirms" or provides "graphic, obvious proof" of a dogma or general conception that is stuck in the head or, in many cases, simply what conforms to a narrow egoistic "interest."

"Abstract thinking" is no merit, as people sometimes think it is, associating the term with the idea of "higher science" as a system of ultra-incomprehensible "abstractions" that hold sway somewhere up above the clouds. This idea of science is held only by those whose sole experience of science is secondhand, who know the terminological surface of the scientific process but have not penetrated to its essence.

Science—if it really is science and not a system of quasi-scientific terms and phrases—is always an expression (reflection) of real *facts*, understood in their interconnection. A "concept"—unlike a term, which requires simply to be memorized—is a synonym for an *understanding* of the essence of facts. A concept in this sense is always concrete, in the sense of object-related. It grows out of facts, and only in facts and through facts does it have sense, "meaning," or content.

Such too is the thinking of the mathematician, which is unintentionally insulted by those who wish to praise it by calling it "abstract." Only the terminological attire of "concepts," only the language of mathematics is "abstract" in mathematical thinking. And if out of the whole of mathematics a person has mastered only its "language," this means that he has mastered it abstractly. In other words, not understanding and not seeing its real object and not knowing how to move independently in accordance with its strict logic—*not seeing reality* from the specifically mathematical point of view, but seeing only the signs that designate it. Perhaps he has also learned some "obvious examples" that illustrate the "application" of these signs.

The real mathematician—like the physicist, like the biologist, like the historian—thinks in fully concrete fashion. He too focuses not on abstract flourishes but on reality itself; only he does so from the special angle or aspect that is specific to mathematics. It is this skill of seeing the surrounding world from the point of view of quantity that constitutes the special feature of the mathematician's thinking.

The person who does not know how to do this is not a mathematician but merely an enumerator and calculator who performs standardized auxiliary operations but is not engaged in the development of mathematical science.

And knowing how to train a mathematician—that is, a person capable of *thinking* in the field of mathematics—is far from the same thing as knowing how to teach a person to count, calculate, and solve "typical tasks." Our schools, alas, are more often oriented toward the latter. Because that is "easier." And then we ourselves start to bemoan the fact that people "capable" of mathematical thinking are such a rarity—one or two in forty. Then, astonished at their "natural talent," we start to "select" them artificially, isolate them from the "untalented masses," and inculcate in them a repulsive self-conceit, the pride and arrogance of the "select" few.

Mathematics as a science, however, is not a whit more complicated and difficult than the other sciences, which do not seem so mysteriously abstract. In a certain sense, mathematical thinking is even simpler and easier. This is evident if only from the fact that mathematical "talents" and even "geniuses" develop at an early age—an age by which it is impossible in other sciences even just to reach the forefront. Mathematics requires *less* and simpler "experience" of the surrounding world than do political economy, biology, or nuclear physics. That is why we do not encounter fifteen-year-old "geniuses" in these fields of knowledge.

If until now we have obtained from our schools a relatively small proportion of people "capable" of mathematical thinking, that is not because Mother Nature is so niggardly in giving out mathematical abilities but for quite another reason.

It is, above all, because we often lead the young child into the sphere of mathematical thinking "upside down" or "back to front." From his very first days at school, we drum into his head "ideas" of mathematical concepts that do not help but, on the contrary, hinder him from *seeing*, from looking correctly at the world around him from the strictly mathematical point of view, which is unfamiliar to him.

The few children who turn out to be "capable" are those who by a fortunate combination of chance circumstances contrive to look out the "window" boarded up by the planks of false ideas. In some places "chinks" remain between these planks, and the curious child sometimes peers through these chinks. And turns out to be "capable."

And these false ideas of elementary mathematical concepts are organically connected with those antiquated philosophical-epistemological ideas about concepts in general and about the relations between these concepts and reality outside thinking, which scientific philosophy abandoned long ago.

Philosophical-logical analysis of the first pages of the arithmetic textbook that introduces the first grader to the realm of mathematical concepts demonstrates this fact beyond dispute. It inculcates in the child an absolutely false (from the point of view of mathematics itself) idea of *number*.

How does the textbook convey to the child the "concept" of number, this fundamental and most general foundation for all his subsequent steps in the field of mathematical thinking?

On the first page there are drawings, very natural and graphic, of a ball and next to it a little girl, an apple (or cherry), a thick stroke (or point), and, finally, the figure "1."

On the second page we find two dolls, two little boys, two melons, two points, and the figure "2." And so on—right up to ten, the "limit" set for the first grader by didactics in accordance with his age-related ("natural") capabilities.

It is assumed that by "mastering" these ten pages the child "masters" the skill of counting and, at the same time, "the concept of number."

In this way he does, indeed, learn how to count. But as for "the concept of number," without realizing it the child swallows instead a quite *false idea* of number—an idea of this most important concept that is even worse than those philistine prescientific ideas with which he comes to school. And a little later this false idea will greatly hinder him in mastering more difficult steps on the path of mathematical thinking.

After mastering the aforementioned pages, the first grader, were he to possess the necessary analytical ability, would answer the question: "What is number?" roughly as follows.

Number is a name that expresses the general abstract property that *all single things* share. The first figure of the series of natural integers is the *name of a single* thing, the figure "2" is the name of "two" single things, and so forth. A single thing is a thing that I see in space as sharply and distinctly demarcated, "cut off" by its contour from all the rest of the world surrounding it—whether the contour is that of a ball, an escalator, a little girl, or a bowl of soup. It is not for nothing that in order to check whether or not the child has mastered this wisdom the teacher shows him an object (it does not matter what kind of object) and asks "how many?" in the hope of hearing "one" in reply. And similarly for two, three, and so on.

But it is self-evident that anyone with the least competence in mathematics will laugh to hear such an explanation of "number" and rightly regard it as childishly naive and false.

In fact, this is merely a *special case* of the numerical expression of reality. And the child is forced to master it as the *most general* case, as an idea of "number in general."

As a result, his very first steps in the realm of mathematical thinking, which he hesitantly takes under the teacher's supervision, already lead him into confusion, into a dead end. It soon turns out that the single object that he is shown is not necessarily called by the word "one": it may be "two" (two halves) or "three" or "eight" or something else. It turns out that the number "1" is anything you like except the name of a single "thing" perceived by the senses. So what is it? What kind of reality do numerical signs designate?

Now even the child who possesses the subtlest and most brilliant analytical abilities will be unable to tell you this. And he will be unable to tell you because two mutually exclusive ideas of number have been deposited in his head and he is unable to relate or "mediate" them. They simply lie "alongside" one another, like two stereotypes, in his "second signal system."

This is very easy to demonstrate by bringing them into open contradiction, into "error."

Show him a toy train consisting of three carriages and a steam locomotive. How many?

One (train)? Four (component parts of the train)? Three and one (the carriages and the locomotive)? Sixteen (wheels)? Six hundred and fifty-four (grams)? Three fifty (the price of the toy in the store)? Half (of the complete set)?

Here we see all the cunning in the abstract question "how many?" to which he has been trained to give a thoughtlessly abstract answer without clarifying "how many what?" And he is even trained to renounce such a wish to clarify, if he had it, as a wish that it is necessary to leave outside the entrance to the temple of mathematical thinking, where in contrast to the world of his direct experience both a tasty candy and a revolting spoonful of castor oil mean "the same"—namely, "one."

The child is "coached" toward this abstraction by the first pages of the "counting" book, which train him completely to divert his attention from any qualitative properties of "single things," to accept that in mathematics lessons "quality" in general has to be forgotten for the sake of pure quantity, for the sake of number, although this is beyond the reach of the child's *understanding*. He can only take it on faith: such, apparently, is the custom in mathematics, in contrast to real life, where he continues to distinguish between candy and castor oil.

Let us suppose that the child has firmly "mastered" the idea of "number" and "counting" explained above and accepts that three melons are "the same" as three pairs of booties—"three" without further clarification.

But now he is let in on a new mystery. Three arshins cannot be compared with three poods:* they are "not the same." Before you can "compare" things—place them on the same numerical scale—you have to make sure that you are dealing with things of the *same name* (same quality). Only "nameless numbers" can be thoughtlessly added and subtracted. A new stereotype, directly opposed to the old one. But which of them should be "applied" or "activated" *in a given case*?

Why is it possible and necessary to "compare" two boys and two cherries in one case, while in another it is not necessary and not permitted? Why in one

*Arshin: an old measure of length, equivalent to about 28 inches or 71 centimeters; pood: an old measure of weight, equivalent to about 36 pounds or 16.4 kilograms.—Trans.

case is this "the same"—namely, single sensually perceptible things without further clarification, while in another "not the same"—*differently named*, *unlike* (though also single) things?

Indeed, why?

The teacher does not explain. He simply shows, using "obvious examples," that in one case you have to do it this way while in the other you have to do it that way. The child is thereby given two highly abstract set ideas of "number" but is not given a concrete *concept*—that is, an *understanding*—of it.

This is very reminiscent of the didactic principles for "learning some sense" that are ridiculed in a wise folktale.

> "Simpleton, oh simpleton, instead of lying on the bed* why don't you go hang around people and learn some sense?"
>
> Catching sight of some peasants hauling sacks of wheat, the obedient and diligent simpleton goes and hangs around one of them, then another. . . .
>
> "Simpleton, you simpleton, you should have said—'I want to join you, not steal from you!'"
>
> The simpleton obediently follows this precious instruction as well.

Here too, the teacher supposes that "concretely"—with the aid of the very obvious expression "hang around"—he has explained to the child how the child can "learn some sense."

But the child, like the simpleton in the tale, does not understand the wise allegories of the adults. He understands them literally, comprehending in their words and explanations only what is familiar and understandable to him from his own life experience. And as his experience is much poorer than the experience of adults and the words that express this experience, he catches only part of the meaning embodied in these words, understanding them literally, *abstractly*—that is, in a one-sided, very general fashion. As a result, instead of acquiring a concrete understanding (and under guise of such an understanding) he learns and takes as his guide an extremely abstract and general (and therefore cunningly ambiguous) prescription. The same is true with regard to "number."

First it is explained to the student that number (one, two, three, etc.) is a verbal or graphical sign that expresses the common property shared by all single things perceived by the senses, no matter what they may be—little boys or apples, iron weights (poods), or wood laths (arshins).

But when he diligently sets about acting on the basis of this abstract idea of number (here as elsewhere, "abstract" does not mean "not obvious"; on the contrary, it is extremely obvious; "abstract" here means poor, meager, one-sided, undeveloped, too general, as "general" as the expression "hang

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*Literally, *pechka*—the heated sleeping platform in a peasant hut.—Trans.

around") and starts to compare poods with arshins, he is reproached: "You are incapable! Here you should have checked first whether these are identically named things."

The diligent and obedient student is prepared to compare only identically named things. But that was not the case here. In the very first task he encounters not just "boys" and not just "apples," but boys mixed up with apples, not to mention pernicious girls each of whom wants to obtain more for an apple than each boy.

It turns out that it is not just possible but even necessary to add, subtract, multiply, and divide numbers that express differently named things—to divide apples by boys, add boys to girls, divide kilograms by meters, and multiply meters by minutes.

Numbers that are identically named in one case and in one sense turn out to be differently named in a second and third case or sense. In one case one stereotype must be activated, in another the directly opposed stereotype. Which of the two should be applied in a given case? Which of the memorized rules has to be recalled? And there are more and more "rules" as you proceed. And they are all contradictory.

And the confused child starts to act by the method of "trial and error," bustling about here and there. When this highly vaunted and unproductive method finally leads him into a dead end and refuses to yield answers coinciding with those printed at the end of the book, the child starts to get nervous, cries, and eventually collapses either into hysteria or into the state of torpid gloom and quiet despair known as the ultraparadoxical phase.

Every one of us, alas, has observed and observes this picture every evening in almost every apartment. Have you really counted the bitter tears shed by young children over their arithmetic homework? But then it is well known that many children experience arithmetic lessons as hard labor or even as a cruel torment, thereby acquiring a lifelong aversion to the subject. In any case, such children outnumber those fortunate individuals—the "able, talented, gifted"—who find in arithmetic an interesting pastime, a field for the exercise of their creative powers, inventiveness, and resourcefulness.

And nature bears not the slightest blame for this situation.

Didactics is to blame. The blame lies with those ideas about the relation of "the abstract to the concrete," of "the general to the single," of "quality to quantity," and of thinking to the world perceived by the senses that to this day, alas, lie at the foundation of many didactical programs.

Elementary analysis of the first pages of the arithmetic textbook described above shows that ideas about all of these logical categories remain at the level of development of logic as a science that this esteemed science had reached at the time of Jan Amos Komensky [Comenius] and John Locke.

The idea of the "concrete" as what is obvious to the senses—an idea that leads in practice to the child having the "abstract" drummed into his head under the guise of the "concrete." The idea of "quantity" (number) as something that is obtained by completely abstracting from any and all "qualitative" characteristics of things, by equating boys with poods and apples with arshins and not, as the science of logic demonstrated over 150 years ago, by analyzing a clearly manifested quality. The idea of a concept as a word or term that expresses the general abstract essence that exists "in all things" of a given kind—a superficial idea that leads to the child mastering merely an *abstract verbal conception* instead of (and under the guise of) a *concrete concept*. The idea of a "contradiction" as something "bad" and "intolerable," as merely an indicator of slovenly and inexact thinking, as something that must be eliminated as fast as possible by means of verbal "clarification" and terminological manipulation.

These are all ideas that today, from the point of view of contemporary logic, from the point of view of dialectics as the logic and theory of knowledge of contemporary materialism, must be evaluated as superficial, archaic, naive, and—let us not beat about the bush—reactionary.

In order that our schools should be capable of teaching how to think and in order that they should actually teach how to think, we must resolutely reconstruct the whole of didactics on the basis of the contemporary—Marxist-Leninist—understanding of all logical categories—that is, of concepts that express the true nature of developing thinking. Otherwise all talk of improving didactics will remain merely a pious wish, and the teaching process based on this didactics will continue to produce "capable minds" only as an exception to the rule. Otherwise we shall continue to place all our hopes concerning the "gifted" on the favors of Mother Nature. We shall wait for these rare favors instead of grasping them.

And a gleam of hope in this regard is already visible.

In the laboratory of the Institute of Psychology of the Academy of Pedagogical Sciences of the RSFSR, research has started under the leadership of D.B. Elkonin and V.V. Davydov specially aimed at laying under the pedagogical process a firm foundation of contemporary philosophical-logical ideas about "thinking" and its connection with "contemplation" (with "obviousness"), the connection between the "universal" and the "single," between the "abstract" and the "concrete," between the "logical" and the "historical," and so on.[1]

In this research an attempt is being made to organize the individual assimilation of scientific knowledge in such a way that it should reproduce in compressed and abridged form the real process of generation and development of this knowledge. Here the child is from the very start not a consumer of set results embodied in abstract definitions, axioms, and postulates but, so to speak, a "co-participant" in the creative process.

By no means, of course, does this mean that each child is forced independently to "invent" all those formulas that people of past generations have already invented for him over the centuries and millennia. But he must retrace the logic of the road traveled. Then he will master these formulas not as abstract magical prescriptions but as real, quite concrete general principles for solving real concrete tasks.

"Concrete general principles"—this sounds rather paradoxical to a person accustomed to thinking (more correctly, to saying) that "general" means "abstract" and that "concrete" means "single" and obvious to the senses.

However, from the point of view of the concepts of dialectics, this is by no means a paradox, by no means an unexpected combination of mutually exclusive terms. From the point of view of dialectics, a concept is precisely "concrete-universal," in contrast to the "abstract-general" term that expresses a one-sided, albeit highly obvious, idea about things.

Thus, researchers at the laboratory of Elkonin and Davydov are convinced that the accepted methodology of teaching how to count (as described above) gives children not the concept of number but merely two abstract and mutually contradictory ideas of number, two special cases of the numerical expression of real things instead of a truly general principle. Moreover, this methodology presents one of these special cases as "general" and the other as more complex, as "concrete."

In one case number expresses the quantity of *single things,* in the other case the quantity of their *component parts*.

Having grasped this, the researchers concluded that the sequence of the accepted methodology must be reversed. First, children should have the truly general nature of number explained to them, and only then should they be shown the two "special cases" of application of the concept.

But clearly you cannot convey to a child the "concept of number" purified of all traces of "obviousness," of connection with any one "special case." So it is necessary to seek and find a "special" (and therefore obvious, sense- and object-related) case in which number and the need for actions with number appear to the child *in general form*. We have to look for a "special" characteristic that expresses only the "general" nature of number and does not again palm off as general something that is merely "special."

Trying to solve this partly psychological, partly logical and mathematical problem, the researchers concluded that it is wrong to start teaching children mathematics with "number"—that is, with counting and computing operations, whether on "single things" or on their "component parts."[2]

There is every reason to suppose that the actions with "numbers" that make up traditional "arithmetic" are far from the simplest and easiest to learn, and that arithmetic does *not* constitute the "bottom floor" of mathematical thinking. The bottom floor rather consists of certain concepts that are usually considered part of "algebra."

Another paradox, for according to long-established tradition "algebra" is a more complex and difficult thing than "arithmetic": only when children reach the sixth grade are they capable of tackling it, and in "the history of mathematics" it came later than arithmetic.

Analysis shows that in the history of knowledge "algebra" must have arisen no later than "arithmetic." Of course, I am talking about the *real* history of people's mathematical development, and not about the history of mathematical treatises, which reflected true history only in retrospect and therefore upside down.

As research demonstrates, man became aware of the very simple quantitative relationships that "algebra" describes before he "invented" number and counting. Indeed, people of necessity must have used such words as "more," "less," "farther," "nearer," "then," "previously," "equal," and "unequal" before they invented number, counting, and the addition, subtraction, multiplication, and division of numbers. It was precisely in these "words" that *general* quantitative (spatial-temporal) relationships between things, phenomena, and events found their expression.

But, naturally, this stage in the development of mathematical thinking was not recorded in special treatises on mathematics. And if the real history of the development of mathematical thinking began before the appearance of the first theoretical treatises on mathematics, then the "logical" sequence for teaching mathematics—that is, for developing mathematical ability—must also start from the real "beginning."

It must start by orienting the child correctly on the quantitative plane of reality, and *not by teaching him number,* which is merely a late (and therefore *more complex*) *form of the expression of quantity,* merely a *special case* of "quantity."

Therefore, it is necessary to start with actions that *mark out* for the child this "quantitative" plane of the reality he sees around him, in order to approach "number" at the next stage as the developed *form of expression* of "quantity," as a later and more complex intellectual abstraction.

The principle of the coincidence of the "logical" with the "historical" is a great principle of dialectical logic. But, once again, its application hangs on one dialectically cunning detail—namely, the logic must correspond to the real history of the object, and not to the history of theoretical ideas about this history.

Analyzing the history of political economy, Karl Marx noted a circumstance that is of great importance from the point of view of dialectics. "The historical development of *all sciences* leads to their *true point of departure* only through a multitude of roundabout and intersecting paths. Unlike other architects, science not only draws castles in the air but erects some stories of the building *before laying its foundation*" (*Contribution to the Critique of Political Economy*, p. 46 [retranslated from Russian]).

Yes, science "discovers" in its object the true "logical foundation" upon which the upper stories rest only in retrospect.

Until then, this "foundation" is *presupposed* by the "upper stories," but is not clearly understood, demonstrated, and analyzed. It is presupposed in a confused, indistinctly formulated fashion, often in the form of "mystical" ideas. This is what happened, for instance, in the case of differentiation. Newton and Leibnitz "discovered" differentiation and taught people how to use it, but themselves could not understand on what real foundations its entire complex construction rests—that is, which "simpler" concepts and actions it really presupposes. This was established only later, by Lagrange, Euler, and other theorists.

Number and counting really presupposed and presuppose as real preconditions a number of ideas that mathematics (like "all sciences") was to come to understand only in retrospect. I speak here of the general preconditions of both number and counting, of the concepts that must be developed (and mastered) before number and counting because they are more general in character and therefore logically simpler.

The mathematical "signs" with the aid of which these simplest and most general concepts are recorded are not figures but rather signs that have long been used by algebra.

They are the signs for equality (=) and inequality (≠) for "more" (<) and for "less" (<). And all of these signs designate *relations between magnitudes*. Precisely between "magnitudes"—that is, between *any* magnitudes, of whatever kind, whether expressed in terms of number or not, whether spatial-geometric or temporal. Relations between *magnitudes in general*.

It is self-evident that the idea of "magnitude" arose in the history of people's thinking before the ability to measure these magnitudes precisely by one means or another and to express them in terms of "number."

The ability specially to mark out from the entire diversity of qualities of things that are perceived by the senses just one quality—namely, their "magnitude." And then the ability to compare these "magnitudes" or to compare things *only as magnitudes*. To judge whether they are *equal* or not. To judge which of them is "bigger" or "closer" and which "smaller" or "farther"—in space or in time.

And then, when it was discovered that judgments of this kind are too "general," too incomplete (= "abstract") to act in the world on their basis, the question began to arise: "bigger" or "smaller" *by how much*? And only at this point did the need for and practice of "number" and "counting" arise.

Because without them, without these more concrete (complex, developed) concepts of quantity it would have been impossible to solve more complex and concrete object-oriented practical tasks connected with reflection of the quantitative determinacy of the surrounding world.

Man "invented" number *not* by "abstracting" from all and any "qualities," not by learning "not to pay attention" to the difference between a stone and a piece of meat, between a stick and a fire. Just the reverse: in "number" and "counting" he found a means for the deeper and more concrete expression precisely of qualitative (the most important and the first) determinacy.

Man's "need" of number arose when and only when life placed him before the necessity of saying to someone else (or to himself) not simply "more" or "less" but *how much* "more" or "less."

This presupposed a higher and more developed way of relating to the things of the surrounding world than that on the basis of which he had learned to distinguish "magnitudes" only approximately—abstractly.

Number presupposes *measure* as a category more complex than "quality" and "quantity"—a category that makes it possible to reflect the quantitative aspect of the quality marked out *more precisely* (more concretely) than before. And to record this more concrete idea precisely with the aid of figures and not simply by means of the words "more," "less," "equal," and "unequal."

From a general, diffuse, and undifferentiated idea of "quantity" man moved toward and arrived at a more perfect and precise—that is, concrete—idea of quantity—namely, "number."

And therefore "number" had a quite concrete—that is, object-related and practical—meaning and significance for man from the very start. And it was a true *concept* of number, even though it had not yet been analyzed theoretically by a single professional mathematician. This did not happen until much later, when not just mathematical thinking but its theoretical "self-consciousness" had come into existence. Initially this self-consciousness took distorted mystical form, as among the Pythagoreans. And it would be many millennia before mathematics reached a true theoretical understanding of number.

It is, evidently, from this true beginning and in this true historical sequence, which mathematics as a science was to discover only in retrospect, that the logical development of the child's mind in the field of mathematics should proceed.

First the child must be taught to orient himself in the most general and abstract fashion on the quantitative plane, to master the most general and abstract relations among things as "magnitudes," and to record these relations on paper with the aid of the signs for "more," "less," "equal," and "unequal."

Here, however, the child learns to orient himself on the plane of quantity *not* by means of "abstract reasoning" but from real situations that are understandable to him—by "evening out" sticks and "matching" nuts with screws, boxes with pencils, and so on. For the child, this is understandable and interesting.

For the child's mind, this is training in the skill of *independently marking out the quantitative-mathematical aspect of real things* in the world of diverse qualities that surrounds him.

And not training in the skill of repeating in parrot-like fashion the word "one" when a single sensually perceived "thing" is shoved in front of his nose, or the word "two" when two such things are shoved there.

Thanks to this, the child, when he is shown one (two, three, etc.) single sensually perceived thing, will no longer reply thoughtlessly to the provocative and abstract question "how many?" with the word "one" ("two," "three," etc.). He will first ask: "How many what?"

And this indicates that he is already—in the case of number—thinking *concretely*. And not like the market trader who thoughtlessly hangs the label of a verbally embodied abstraction upon a concrete thing and thinks that her "understanding" of this thing is thereby complete.

If to his legitimate question the child receives the answer: "I am asking how many things there are here," he will reply with confidence and precision: "One."

If it is explained to him: "I am asking how many centimeters," he will reply: "two," "about two," or say: "It has to be measured." He understands that expression in terms of a number (a figure) requires measurement, measure.

Two important elements of "intelligence" are trained here simultaneously. First, the ability to relate correctly to a question ("how many?") and to ask a question oneself to clarify the task in terms sufficiently concrete to make possible a precise and unambiguous answer ("how many *what*?"). And second, the ability correctly to correlate a numerical sign *with reality* in its mathematical aspect.

The child's mind proceeds here not from obvious particulars to the abstractly general, because this is a quite unnatural and fruitless path in science, but from the truly universal (abstract) to the diversity of particulars within his grasp (i.e., to the concrete).[3]

For this is how science itself develops, assimilating more and more "particulars" in the light of initial principles. And not the other way round, not departing from "particulars" into the lofty heights of empty abstraction.

Here thinking moves constantly within sense- and object-related (and therefore also "obvious") material, moves in accordance with facts, never for a moment severing connection with them.

In this way the child masters the actual *sense- and object-related reality of mathematical concepts,* and not a poor ersatz substitute for that reality, not "obvious examples" of set abstractions that are incomprehensible to him. Mathematical thinking develops within him. There is no need to cram into him heaps of abstract words, prescriptions, stereotyped schemas, and "typical solutions" that he is then quite unable to "apply." Therefore, he does not then face the supremely absurd task of somehow "applying" the general knowledge he has acquired (i.e., crammed) to life, to reality. For him this general knowledge is, from the very start, none other than reality itself, reflected in its essential features—that is, in concepts. In concepts he masters precisely the *reality* that they reflect. And not "abstractions" that he is then quite incapable of correlating with "reality."

The reader-pedagogue who hoped to find in this article a detailed set prescription in answer to the question "How should I teach how to think?" will probably be disappointed. All this is too general, he will say, even if it is true.

Quite right. Philosophy is incapable of offering the pedagogue any set prescriptions or "algorithms" on this score. A great deal more effort must be expended in order to bring the principles I have enunciated to such a degree of concreteness as would make them directly applicable to daily pedagogical practice. This will require collaborative efforts on the part of philosophers and logicians, psychologists, specialists in mathematics and in history, and, of course, pedagogues themselves.

Anyone who wants to teach others how to think must himself know how to think. You cannot teach someone else to do what you do not know how to do yourself.

No didactics will teach a pedagogue how to teach thinking if that pedagogue is an indifferent machine-like person accustomed to working in accordance with stereotype, to following a rigidly programmed algorithm in his head. Each pedagogue must be able to apply general theoretical—and, in particular, general philosophical—principles to his own concrete subject. He should not wait for someone else to do this for him and bring him a collection of set prescriptions that relieve him of the burden of intellectual labor, of the need above all to do his own thinking. Even the best and most elaborately developed didactics will not free the pedagogue of this necessity. However concrete and detailed it may be, between its general propositions and unique pedagogical situations there will remain a gap. And only the pedagogue who thinks dialectically, only the person with a developed "power of judgment" will be able to overcome this gap (between the "universal" and the "single").

Our schools must teach how to think. This means that each pedagogue must teach how to think. To think at the level of contemporary logic—that is, at the level of dialectics as the logic and theory of knowledge of the materialism of Marx, Engels, and Lenin. Without this all our efforts will come to naught, and didactics will remain at the level of John Locke and Jan Amos Komensky.

Notes

1. See V.V. Davydov, "Sviaz′ teorii obobshcheniia s programmirovaniem obucheniia," in *Issledovaniia myshleniia v sovetskoi psikhologii* (Moscow: Nauka, 1966).

2. For a detailed analysis of this problem, see the book *Vozrastnye vozmozhnosti usvoeniia znanii (mladshie klassy shkoly)*, ed. D.B. El′konin [Elkonin] and V.V. Davydov (Moscow: Prosveshchenie, 1966).

3. For a more detailed treatment see, for example, my book *Dialektika abstraktnogo i konkretnogo v "Kapitale" K. Marksa* (Moscow: Izd-vo AN SSSR, 1960).